Matrix : Matrix is a rectangular arrangement of numbers arrange in some rows and columns. Generally a matrix enclose by **( )** or **[ ]** or **| |.** The number of rows and columns is called the order of the matrix.
Example:

(i) Let A be defined by the following matrix with order $m \times n$ i.e. m rows and n columns

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ \vdots \\ m \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & \dots & n \end{array} \\ \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ a_{31} & a_{32} & \dots & a_{3n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} \end{array}$$

(ii) $A = \begin{bmatrix} 1 & 3 & 2 \\ 8 & 3 & 3 \\ 2 & 3 & 5 \end{bmatrix}_{3\times 3}$ is a matrix of order 3×3 i.e. containing 3 rows and 3 columns.

(iii) $A = \begin{bmatrix} 1 & 2 & 4 \\ 8 & 3 & 6 \end{bmatrix}_{2\times 3}$ is a matrix of order 2×3 i.e. containing 2 rows and 3 columns.

Types of Matrices:

Square matrix

A matrix is said to be a square matrix if the number of rows is equal to the number of columns.

For example, $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ is square matrix containing 3 rows and 3 columns.

Rectangular Matrix

A matrix is said to be a rectangular matrix if the number of rows is not equal to the number of columns.

Example, A is a matrix of the order 3 × 2. $A = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 2 \end{bmatrix}$

Diagonal matrix

A square matrix A is said to be a diagonal matrix if all its non-diagonal elements are zero. For example, $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

Zero or Null Matrix

A matrix is said to be zero matrix or null matrix if all its elements are zero.

For Example, $A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Upper Triangular Matrix

A square matrix in which all the elements below the diagonal are zero is known as the upper triangular matrix. For example, $A = \begin{bmatrix} 2 & 2 & -1 \\ 0 & 5 & -4 \\ 0 & 0 & 3 \end{bmatrix}$

Lower Triangular Matrix

A square matrix in which all the elements above the diagonal are zero is known as the upper triangular matrix. For example, $A = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 5 & 0 \\ 1 & 3 & 3 \end{bmatrix}$

Transpose of a Matrix:

The transpose of a matrix is simply a flipped version of the original matrix. We can transpose a matrix by switching its rows with its columns. We denote the transpose of matrix A by A^T. For example, if $A = \begin{bmatrix} 0 & 2 & -1 \\ -2 & 0 & -4 \\ 1 & 4 & 0 \end{bmatrix}$ then $A^T = \begin{bmatrix} 0 & -2 & 1 \\ 2 & 0 & 4 \\ -1 & -4 & 0 \end{bmatrix}$

Symmetric matrix.

A matrix can be symmetric only if it is square. If the transpose of a matrix is equal to itself, the matrix is said to be **symmetric**. This means that for a matrix to be symmetric, $A^T = A$

$A = \begin{bmatrix} 1 & 5 & 22 \\ 5 & 2 & 13 \\ 22 & 13 & 3 \end{bmatrix}$ then $A^T = \begin{bmatrix} 1 & 5 & 22 \\ 5 & 2 & 13 \\ 22 & 13 & 3 \end{bmatrix}$ So, $A^T = A$, which is **symmetric**.

Examples:

$A = \begin{bmatrix} 1 & 5 & 6 \\ 5 & 2 & 11 \\ 6 & 11 & 7 \end{bmatrix}$	$A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 2 & -4 \\ -1 & -4 & 3 \end{bmatrix}$	$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 2 & 4 \\ 1 & 4 & 3 \end{bmatrix}$

Anti-symmetric Matrix/ Skew-symmetric matrix

A matrix can be skew symmetric only if it is square. If the transpose of a matrix is equal to the negative of itself, the matrix is said to be **skew symmetric**. This means that for a matrix to be skew symmetric, $A^T = -A$.

Let, $A = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$ then $A^T = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} = -A$

So, $A^T = -A$, which is **skew symmetric**.

For **examples:**

(i) $A = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$	(ii) $A = \begin{bmatrix} 0 & 2 & -1 \\ -2 & 0 & -4 \\ 1 & 4 & 0 \end{bmatrix}$
(iii) $A = \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}$	(iv) $A = \begin{bmatrix} 0 & -2 & 3 \\ 2 & 0 & -5 \\ -3 & 5 & 0 \end{bmatrix}$
(v) $A = \begin{bmatrix} 0 & 2 & 3 \\ -2 & 0 & -4 \\ -3 & 4 & 0 \end{bmatrix}$	(vi) $A = \begin{bmatrix} 0 & -2 & -5 \\ 2 & 0 & -3 \\ 5 & 3 & 0 \end{bmatrix}$